MATH 2260 Calculus II
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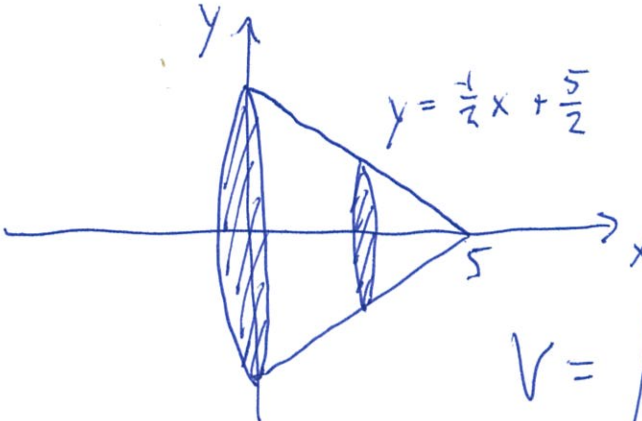
Name: Answers
Date: June 18, 2015

Quiz No. 1

Show all of your work, label your answers clearly, and do not use a calculator.

(25) **Problem 1** *Find the volume of the solid generated by revolving the triangular region in the first quadrant below the line $x + 2y = 5$ about the x-axis.*



$$x + 2y = 5$$
$$2y = 5 - x$$
$$y = -\frac{1}{2}x + \frac{5}{2}$$
(5)

$$V = \int_0^5 A(x)\,dx = \int_0^5 \pi\left(+\frac{1}{2^2}x^2 + 2\frac{5}{2}\left(\frac{-1}{2}\right)x + \frac{25}{4}\right)$$

(15)

$$= \pi\left[\frac{1}{4}\left(\frac{x^3}{3}\right) - \frac{5}{2}\left(\frac{x^2}{2}\right) + \frac{25}{4}x\right]_0^5$$ (5)

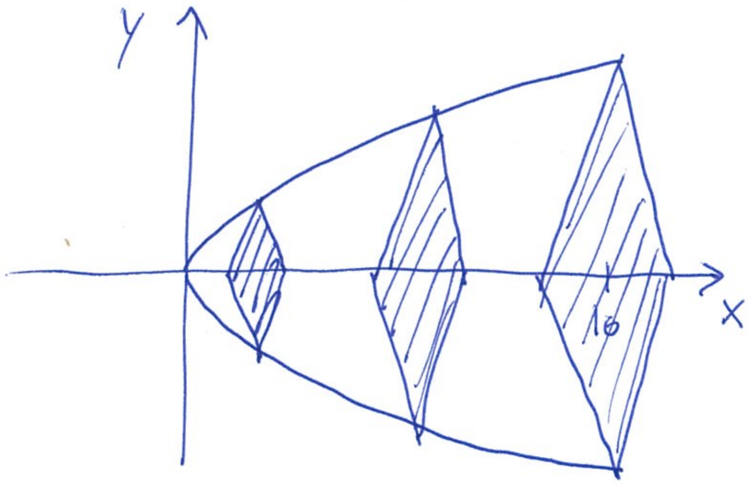
$$= \frac{\pi}{4}\left(\frac{5^3}{3} - 5^3 + 5^3\right)$$

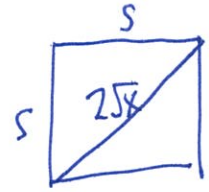
$$= \frac{5^3\pi}{12} = \frac{125\pi}{12}$$

-5 for incorrect substitution

-5 for $(a+b)^2 = a^2 + b^2$
-3 for addition to multiplication
-1/2 for arithmetic errors that don't affect process.
-3 Wrong axis, wrong variable
-1 for no dx's
-3 for arithmetic errors that do affect process.
-3 for misuse of equals
-3 for wrong bounds

(25) **Problem 2** *Find the volume of the solid described below: The solid lies between the planes perpendicular to the x-axis at $x = 0$ and $x = 10$. The cross-sections perpendicular to the axis on the interval $0 \leq x \leq 10$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$.*





$$s^2 + s^2 = (2\sqrt{x})^2$$

$$2s^2 = 4x$$

$$s = \sqrt{2x}$$

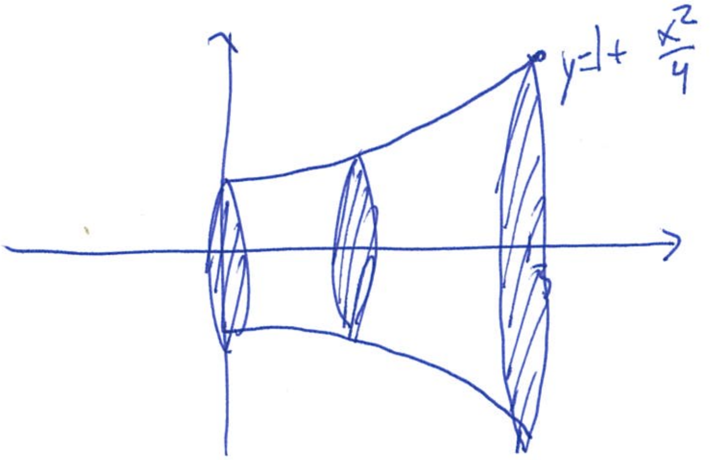
$$\Rightarrow A(x) = s^2 = 2x$$

$$V = \int_0^{10} A(x)\,dx = \int_0^{10} 2x\,dx = 2\left[\frac{x^2}{2}\right]_0^{10} = 100$$

-5 for confusing length of diagonal and side length

-5 for wrong antiderivatives

(25) **Problem 3** *Find the volume of the solid generated by revolving the region described about the given axis: The region is enclosed above by the curve* $y = 1 + \frac{x^2}{4}$, *below by the x-axis, to the left by the y-axis, and to the right by the line* $x = 3$, *rotated about the x-axis.*



$$V = \int_0^3 A(x)\,dx$$

$$= \int_0^3 \pi\left(1+\frac{x^2}{4}\right)^2 dx$$

$$= \pi\int_0^3 1 + 2\frac{x^2}{4} + \frac{x^4}{16}\,dx$$

$$= \pi\left[x + \frac{1}{2}\left(\frac{x^3}{3}\right) + \frac{1}{16}\left(\frac{x^5}{5}\right)\right]$$

$$= \pi\left(3 + \frac{9}{2} + \frac{3^5}{16\cdot 5}\right)$$

(25) **Problem 4** *Find the length of the curve* $x = \frac{y^3}{3} + \frac{1}{4y}$ *from* $y = 1$ *to* $y = 6$.

$$L = \int_1^6 \sqrt{1 + \left(\frac{\partial x}{\partial y}\right)^2}\, dy$$

$$= \int_1^6 \sqrt{1 + \left(y^2 + \frac{1}{4}\left(-\frac{1}{y^2}\right)\right)^2}\, dy$$

$$= \int_1^6 \sqrt{1 + y^4 - \frac{1}{2} + \frac{1}{16y^4}}\, \delta y$$

$$= \int_1^6 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}}\, dy$$

$$= \int_1^6 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}\, dy$$

$$= \int_1^6 \left(y^2 + \frac{1}{4y^2}\right) dy$$

$$= \left[\frac{y^3}{3} + \frac{1}{4}\left(\frac{y^{-1}}{-1}\right)\right]_1^6 = \left(\frac{6^3}{3} - \frac{1}{24} - \left(\frac{1}{3} - \frac{1}{4}\right)\right)$$

$$= \left(36 \cdot 2 - \frac{1}{24} - \frac{8}{24} + \frac{6}{24}\right)$$

$$= 72 - \frac{1}{8}$$

-20 for no arc length formula

-3 for derivative mistakes

-5 for no completing the square

-10 for attempt at arc length formula